STATISTICS SEMINAR

TITLE: **The Mathematics and Statistics of Manifold Learning**

Speaker:
Tyrus Berry
Assistant Professor of Mathematics George Mason University

Time & Place:
Wednesday, October 9, 2019
4:00, Room 140 Bardeen
Cookies & Coffee @ **3:30**, Rm 1210 MSC

Abstract: Despite fundamental limitations from statistical learning theory (in particular, the curse-of-dimensionality), various empirically driven approaches continue to obtain surprisingly successful results on certain problems. Explaining these results presents an opportunity for the mathematics and statistics communities to learn about the types of structure and regularity that can occur in real data. If these structures can be rigorously identified, optimal methods can be designed to test for and exploit their presence. Manifolds (nonlinear subspaces of the ambient data space) provide a particularly nice mathematical structure for circumventing the curse-of-dimensionality since the intrinsic and extrinsic dimensionality can be decoupled. While manifolds are somewhat rigid structures for real (noisy) data sets, their long history of mathematical study provides us with many useful tools that we may hope to generalize in the future. We call the assumption that a data set is restricted to lie on a manifold the geometric prior. In this talk we will focus on a method of representing an unknown manifold based on a data set by estimating the Laplace-Beltrami operator with a sequence of graph Laplacians. We will overview the mathematical and statistical tools used to prove this convergence (including some new results on manifolds with boundary), and justify the claim that mathematically the Laplace-Beltrami operator contains all information about the manifold. We will then introduce a new method, called the Spectral Exterior Calculus (SEC) that actually uses the Laplace-Beltrami operator to construct this geometric information (known as the exterior calculus on the manifold). We also indicate how this method may ultimately lead to generalizing beyond manifolds to more realistic priors.